# A Disturbance Observer with an Autoregressive Moving Average Method for a One-wheel Robot System

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*Abstract* – This paper presents a nominal model estimation method for a disturbance observer (DOB) used in controlling a one-wheel robot system. The fundamental idea of disturbance observer is to use the inverse model of the plant to identify the disturbance backward from the output. The performance of DOB depends upon the identified model. In this paper, the recursive least square (RLS) method for identifying the nominal plant model is proposed by an autoregressive moving average method (ARMAM). Input-output data is used for the estimation of model parameters of an unknown plant. Experimental studies of balancing a one-wheel robot with two differently induced models for DOB design are investigated and compared

*Keywords* – ARMA model, model estimation, disturbance observer, RLS, one-wheel robot.

## **1. Introduction**

As a powerful robust control method, disturbance observer (DOB) has been presented and used in a variety of motion control application. The basic architecture of DOB is too use the inverse model to extract the disturbance information and use it cancel it out. Therefore, the performance is pretty dependent upon the model accuracy.

There is a way to compensate for the model inaccuracy to improve the performance of DOB. Introducing a Q filter in the DOB loop actually compensates for the discrepancy between the estimated and real disturbance. The general form of the Q filter is the lowpass filter type to satisfy mainly disturbance rejection.

There is another consideration of sensor noise immunity for the Q filter to satisfy. These two requirements are contradicted each other so that there should be a trade-off between the robustness and the sensitivity. There are well known methods for designing Q-filters [1-3].

In general, the procedure of designing DOB follows the steps. Firstly, the estimation of plant model is required. Secondly, design a Q-filter considering the trade-off relation. Thirdly, check the performance. If the performance is not satisfactory, designing Q filters instead of remodeling the plant is repeated. Most of research on DOB design focus on designing Q filters. The Q filter may have the function of compensating modeling errors.

However, designing an appropriate Q filter to satisfy the performance index becomes a time-consuming procedure

and it is quite difficult to find the optimal Q filters. Exact identification of the plant model may reduce the burden of designing the Q filter.

In this paper, therefore, system identification is focused for a one-wheel robot system. System identification is a statistical method where observed data are used to find out the mathematical model of dynamic system. As a time domain method, there are an autoregressive (AR) system which is an infinite impulse response filter, a moving average (MA) system which is a finite impulse response filter, and an autoregressive moving average (ARMA) system which is a combined model of AR and MA.

In general, the adaptive algorithms such as least squares method (LMS) or recursive least squares method (RLS) are used to find out the system parameters by virtue of ARMA model taking advantages of both AR and MA [4-9].

Since the one-wheel robot system is an under-actuated and non-holonomic system that has many constraints in design, balancing control is quite challenging. Force induced by the gyroscopic effect is the main control input to the system. Two actuations such as the rolling angle of the body and the tilting angle of the gimbal are coupled and must be controlled appropriately for the successful balancing performance.

Based on the simple model of an inverted pendulum or stick systems, friction force from the ground is considered as a disturbance to the system [10,11]. Such a system shows marginal stability characteristic in the control performance. Therefore, in this paper, DOB is designed for the one-wheel robot system to reject the disturbance.

In the framework of DOB design, the robot model parameters of a nominal second order system are estimated in the real-time with an autoregressive moving average method (ARMA) where recursive least square (RLS) algorithm is used. The second-order proper function is preferred as an identified model since the inverse of the model is used in the DOB structure.

Finally, experimental studies of balancing control tasks of a one-wheel robot are conducted to confirm the proposed method by comparing the balancing performances of two different DOBs.

## 2. One-wheel Robot System

## 2.1 System Configuration

One-wheel robot system called GYROBO has been developed at Chungnam National University [10-13]. The overall configuration of the system is depicted in Fig. 1. The system can be divided into two subsystems such as a gimbal system and a body system. In the gimbal system, there are a flywheel, a flywheel motor, and a tilt motor. In the body system, there are a driving motor, a gimbal system, and a control system.



Fig. 1. GYROBO Robot system architecture.

The control block diagram is shown as Fig. 2. The main control hardware is a DSP. An AHRS (attitude and heading reference system) sensor for the heading angle measurement and an optical encoder for the tilt angle measurement are utilized in the robot system.



#### **2.2 Control Mechanism**

The main goal of the one wheel robot is to drive in its terrain while maintaining balance. Control mechanism is illustrated in Fig. 3.



Fig. 3. Control mechanism.

The robot is easily to lean toward either side by nature. The tilting of a high-rotating flywheel prevents the robot from leaning one direction. This is known as a gyroscopic effect that generates a force in the yawing direction of the body system. Combining the gyroscopic force with the frictional force accomplishes the lateral motion control in the roll direction. Therefore, the main control input to the system is the tilting motion of the gimbal system.

However, it is found that the tilting angle cannot return to the original position when it tries to maintain balance. It keeps leaning against one direction and this results in GYROBO's falling down. In addition, the tilted position of the flywheel makes a pitch motion as well as a yaw motion. The coupled motion makes a system also unstable.

Therefore, it is important to prevent a tilt angle of the gimbal system from diverging and to make it reside within a certain bound. It is not easy to solve the problem under the disturbed dynamic situations. Here, our proposal is to use DOB control scheme with help of the RLS algorithm to identify the system model more exactly.

#### 3. Model Identification

# 3.1 ARMA system

The robot system is assumed as a second order system which has a proper transfer function form. The proper transfer function can be described as (1) in the continuous time domain.

$$P(s) = \frac{c_0 s^2 + c_1 s + c_2}{d_0 s^2 + d_1 s + d_2}$$
(1)

The IIR filter (1) can be rewritten as a discrete time domain form as follow.

$$P(Z) = \frac{a_0 + a_1 Z^{-1} + a_2 Z^{-2}}{1 + b_1 Z^{-1} + b_2 Z^{-2}}$$
(2)

The corresponding difference equation can be shown as follow.

$$y(n) = a_0 x(n) + a_1 x(n-1) + a_2 x(n-2)$$
  
-b\_1 y(n-1) - b\_2 y(n-2) (3)

(4)

The ARMA model (3) can be briefly described as a recursive way.

 $\mathbf{y}(\mathbf{n}) = \mathbf{u}^{\mathrm{T}}(\mathbf{n}) \boldsymbol{\psi}(\mathbf{n})$ 

where,

$$\Psi^{T}(n) = \begin{bmatrix} a_{0} & a_{1} & a_{2} & b_{1} & b_{2} \end{bmatrix},$$
  
$$\Psi^{T}(n) = \begin{bmatrix} x(n) & x(n-1) & x(n-2) & y(n-1) & y(n-2) \end{bmatrix}$$
(5)

Therefore, the proper transfer function can be transformed as a recursive equation between input and output data. Their coefficients can be estimated by the recursive least squares method (RLSM).

#### 3.2 Review of RLS

Using a parameter estimation method, (4) can be rewritten as follow [14].

$$\mathbf{y}(\mathbf{n}) = \mathbf{u}^{\mathrm{T}}(\mathbf{n})\hat{\boldsymbol{\psi}}(\mathbf{n}) + \hat{\mathbf{e}}(\mathbf{n})$$
(6)

where  $\hat{e}(n)$  is the estimation error. From (6),

$$\hat{\mathbf{e}}(\mathbf{n}) = \mathbf{d}(\mathbf{n}) \cdot \mathbf{u}^{\mathrm{T}}(\mathbf{n})\hat{\boldsymbol{\psi}}(\mathbf{n})$$
(7)

(7) can be rewritten as follow.

$$\hat{\mathbf{e}}(\mathbf{n}) = \mathbf{u}^{\mathrm{T}}(\mathbf{n})(\boldsymbol{\psi}(\mathbf{n}) - \hat{\boldsymbol{\psi}}(\mathbf{n}))$$
(8)

where  $\Psi(n)$  is the actual parameter.

RLS is an algorithm to find out  $\hat{\psi}(n)$  by minimizing an objective function.

$$\mathbf{J} = \sum_{n=0}^{N} \mathbf{\hat{e}}^{2}(n) = \mathbf{\hat{e}}^{T}(n)\mathbf{\hat{e}}(n)$$
(9)

Optimal solution of (9) can be obtained by the gradient

$$\frac{\partial}{\partial \hat{\psi}} \mathbf{J} = 0 \tag{10}$$

$$\hat{\boldsymbol{\psi}}(n) = [\boldsymbol{u}^{\mathrm{T}}(n)\boldsymbol{u}(n)]^{-1}[\boldsymbol{u}^{\mathrm{T}}(n)\boldsymbol{d}(n)]$$
(11)

Covariance matrix can be defined as follows.

$$\mathbf{P}(\mathbf{n}) = [\mathbf{u}^{\mathrm{T}}(\mathbf{n})\mathbf{u}(\mathbf{n})]^{-1}$$
(12)

Using the Matrix Inversion Lemma, we have

$$\mathbf{P}(t+1) = \mathbf{P}(t)(\mathbf{I} - \frac{\mathbf{u}(n+1)\mathbf{u}^{\mathrm{T}}(n+1)\mathbf{P}(t)}{1+\mathbf{u}^{\mathrm{T}}(n+1)\mathbf{P}(t)\mathbf{u}(t+1)}) \quad (13)$$

$$\hat{\boldsymbol{\psi}}(n+1) = \hat{\boldsymbol{\psi}}(n) + \mathbf{P}(n+1)\mathbf{x}(n+1)(\mathbf{y}(n+1) - \mathbf{x}^{\mathrm{T}}(n+1)\hat{\boldsymbol{\psi}}(n))$$
(14)

## 4. ARMA-DOB

## 4.1 DOB

The main goal of the DOB design is to identify the disturbance more accurately and then to cancel it out. The conventional DOB architecture is shown in Fig. 4.

In Fig. 4, u is the command input, v is the control input, d is the disturbance, y is the system output,  $\varepsilon$  is a sensor noise,  $\hat{d}$  is the estimated disturbance, G(s) is the real plant,  $G_n(s)$  represents for the nominal model, and Q(s) is the filter.



The transfer functions from three inputs as follows.

$$G_{uy}(s) = \frac{G(s)}{1 - Q(s) + Q(s)G_n^{-1}(s)G(s)}$$
(15)

$$G_{dy}(s) = \frac{G(s)[1 - Q(s)]}{1 - Q(s) + Q(s)G_n^{-1}(s)G(s)}$$
(16)

$$G_{ey}(s) = \frac{G(s)G_n^{-1}(s)G(s)}{1 - Q(s) + Q(s)G_n^{-1}(s)G(s)}$$
(17)

The output can be represented as

(

$$y(s) = G_{uy}(s)u(s) + G_{dy}(s)d(s) + G_{\varepsilon y}\varepsilon(s)$$
(18)

In (16), the effect by disturbance input can be minimized when  $Q(s) \approx 1$  and  $G(s) = G_n(s)$ .

$$G_{dv}(s) = 0 \tag{19}$$

However, for the sensor noise immunity in (17)  $1-Q(s) \approx 1$  is required when  $G(s) = G_n(s)$ .

$$G_{\rm ev}(s) = 0 \tag{20}$$

Therefore, there is a trade-off of adjusting the design factor Q(s) between (19) and (20). The trade-off between the disturbance and the noise must be considered in the QA filter design. Both conditions of (19) and (20) can be achieved under the assumption of  $G(s) = G_n(s)$ . However, it is not easy to achieve that identifying the proposed system exactly to get the right model is difficult.

To remedy for the difficulty, in the next step, we propose the on-line model parameter tuning DOB method where ARMA model parameters are estimated in the real-time fashion.

## 4.2 ARMA-DOB

In Fig. 5, the proposed ARMA-DOB is described. In the framework of the DOB structure, system model parameters are identified in the real-time fashion. The inverse of the identified model is used directly in the DOB.



### 5. Experimental Studies

### 5.1 Experimental setup

Experimental studies of the balancing control performance of a one-wheel robot system are conducted. The experimental setup is shown in Fig. 6. The DOB based controller is embedded in DSP



Data logger RS232 cable Robot

Fig. 6. Experimental setup.

#### 5.2 Experimental results

Two experiments are conducted. One is the balancing control by the conventional DOB control scheme. Another is by the proposed method. The two results are illustrated in Fig. 7 and 8, respectively.

We see that the robot maintains balance well by both control methods. The deviation of the balancing angle is smaller by the proposed DOB method as shown in Fig. 8 compared with those of Fig. 7. The roll angle is controlled within 1 degree and the tilting angle is controlled within 10 degrees by the proposed DOB while 2 degrees and 30 degrees for the conventional DOB.



Fig. 7. Balancing results of conventional DOB



Fig. 8. Balancing results of the proposed DOB.

# 6. Conclusion

In the paper, we have presented a novel DOB design method with the help of RLS algorithm. The difficulty of model identification has been tackled by the on-line model parameters estimation method of an ARMA system. The proposed method has been implemented in the real system and experimental studies of balancing control of one-wheel robot system were conducted. The robot is successfully maintains balance by both schemes.

However, it is found that the proposed ARMA-DOB control scheme performs better than that of the conventional DOB since the inverse model of the system by ARMA is more accurate in the DOB structure. When the roll angle of the body system and the tilt angle of the gimbal system are compared, the proposed control scheme performs better.

#### Acknowledgement

This work has been supported by the 2015 the basic research funds through the contract of National Research Foundation of Korea (NRF-2014R1A2A1A11049503).

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